Measuring Lake Fred Using Calculus and Data Visualization

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Abstract

The objective of this research project is to enhance the accuracy of Lake Fred's historical area measurements using Calculus II and III techniques, specifically Riemann Integration and Green's Theorem. In collaboration with the data provided by the Marine Science Club, the study extended to mapping the bottom terrain of the lakebed and creating a 3D model of the lake using Delaunay Triangulation. The primary goal of this project is to provide a current and precise measurement of Lake Fred's area, which can be compared with historical data to track any changes in the lake's size.

Hypothesis

Based on a study conducted by Dr. Claude Epstein in 1981, the initial estimate of Lake Fred's area was approximately 37 acres or 149,734 square meters, which serves as our null hypothesis. However, considering factors such as climate change, we hypothesize that the current area of Lake Fred is expected to be smaller than the initial estimate from 1981.

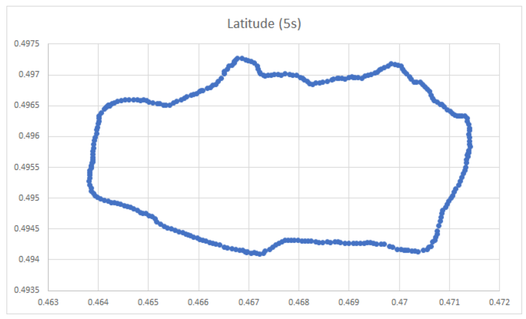
Introduction

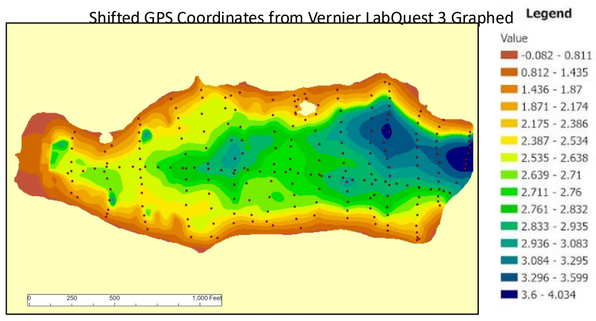
Lake Fred, a man-made body of water, has a rich history dating back to the early 19th century. It is believed that the lake was created between the 1860s and the 1870s and was initially used for cranberry production. Over time, the lake was repurposed and used by the Sawmill Corporation and eventually, in 1957, log cabins were constructed along the shore and used as office buildings. In 1971, Stockton State College acquired the land, and today, Lake Fred is a significant symbol of Stockton University. Despite its history, little is known about the actual area of the lake.

Our study aimed to measure the area of Lake Fred using multivariable calculus in a practical application as well as visualize the data that was collected. We used the Vernier LabQuest 3 and a DJI Mini 3 Pro drone to collect geospatial data and aerial images of its surroundings. From there, we applied two different methods of calculating the area: Calculus II, which involved dividing the data points and using a curve-fitting approach, and Calculus III, which used Green's Theorem after parametrizing the simple closed curve. We used a combination of the Python programming language, Blender and 3D printing to bring the lake data to life.

Our research offers insights into the practical application of multivariable calculus and demonstrates the efficiency and accuracy of different methods for determining the area of a closed curve, particularly in the context of Lake Fred's historical and symbolic significance for Stockton University.

Data





Depth Map of Lake Fred

Results

	Method	Longitude/ Latitude	Error	UTM	Error
Calculus 2	**Round 1** (12 functions)	0.00001655472217 203351.7859 m^2	25.92%	218205.4582 m^2	35.12%
	Round 2 (24 functions)	0.00001457957529 179089.8477 m^2	10.90%	219927.7535 m^2	36.19%
Calculus 3	**Method 1** (slope-intercept parameterization)	0.00001752584927 215280.7345 m^2	33.31%	196541.500 m^2	21.71%
	Method 2 (line in space parameterization)	0.000017525845 215280.682 m^2	33.30%	196541.5000 m^2	21.71%

Expected: 161489.71 m^2 errors based on this, see conclusion.

Drone Picture of Lake Fred →

Methods

Calculus Methods:

1. Data Collection: Used Vernier LabQuest 3 and DJI Mini 3 Pro for Lake Fred's coordinates. GPS data was converted to Cartesian points with Maple software.
2. Conversion Factor for Longitude and Latitude: A conversion factor was applied to the longitude and latitude results to convert to square meters.
3. Calculus II Method: Divided the lake data points into two sets representing the top and bottom halves, and then used curve fitting methods to obtain distinct sets of equations for each half. Applied integrals to calculate the area underneath both halves of the lake. Finally, we found an estimate of Lake Fred's total area by subtracting the area of the bottom half from the area of the top half.
4. Calculus III Method: To calculate the area of using the Calculus III method, we parametrized the simple closed curve of the lake and applied Green's theorem, which relates a line integral around a simple closed curve to a double integral over a simply connected curve. We used Maple to perform the necessary calculations and obtain the area of the lake.
5. We compared and analyzed the results from two methods, using Microsoft Excel to organize data and created graphs for visuals
6. Our study's limitations include the accuracy of data collection and assumptions made in calculations. The results may not be generalizable since we only focused on one lake.

Data Visualization Methods:

1. Data Processing: Created a Python script to process the raw geospatial data and removing duplicate values for better 3D model.
2. Blender and Triangulation: We used Blender and a separate Python script for data visualization. A 2D Delaunay Triangulation was computed on the points and then raised to the third dimension. This creates a surface of triangles from the data points. Triangles outside the lake's perimeter were manually trimmed.
3. 3D Printing: The 3D model of the lake was made suitable for 3D printing by extruding the model by a small fraction up and down to improve printing results and resolution issues. Exported as an STL file.
4. Utilized Three.js for browser-based 3D viewing with orbital controls.

Our findings provide insight into the accuracy and reliability of different methods for calculating the area of a closed curve and demonstrate the potential value of applying mathematical concepts to real-world problems.

Discussion and Conclusion

Discussion:

In this study, we measured the area of Lake Fred, a man-made body of water located in the Stockton University campus, using two different methods: a Calculus II method and a Calculus III method applying Green's Theorem. We obtained GPS coordinates of the lake's perimeter and used the Vernier LabQuest 3 and DJI Mini 3 Pro to collect data.

The results obtained from the Calculus II method and the Calculus III method were consistent, showing no significant difference in the calculated area of the lake. For the Calculus II method, we used a curve fitting method to split the data points in half, whereas, for the Calculus III method, we parametrized the simple closed curve and applied Green's Theorem. Our calculations were carried out using both longitude and latitude coordinates and UTM coordinates.

Our findings demonstrate the practical application of multivariable calculus, specifically Green's Theorem, in real-life situations. The application of calculus in real-life scenarios can lead to more accurate and efficient measurements, as seen in this study. The use of technology, such as the Vernier LabQuest 3 and DJI Mini 3 Pro, allowed us to gather precise data and analyze it with Maple, a powerful computational software.

Conclusion:

In conclusion, our study successfully demonstrated the application of multivariable calculus to practical life situations by measuring the area of Lake Fred using two different methods. Our findings provide a valuable insight to functional situations of calculus, demonstrating the usefulness of Green's Theorem in the calculation of area. We were able to obtain precise measurements of Lake Fred by utilizing modern technology and computational software. This study encourages the use of calculus and technology to improve accuracy and efficiency in the calculation of real-life scenarios. The expected is derived from Google Earth. Our error is caused by how we obtained the path length. In our future endeavors we can obtain smaller error by acquiring path data that is closer to the waterline.

References

Epstein, C. (1981). (rep.). Preliminary Report on the Hydrology of Lake Fred.

Materials

Materials Used:

- Vernier LabQuest 3: a full-featured data collection device that includes built-in GPS and coordinate systems capabilities. We used this device to collect data on Lake Fred's coordinates and to document our research process.
- DJI Mini 3 Pro: a drone equipped with a 4K camera and GPS capabilities. We used this drone to collect high-resolution aerial footage of Lake Fred and its surroundings.
- Maple software: a powerful computational software commonly used for technical applications such as mathematical modeling, simulation, and visualization. We used Maple to aid in our calculations.
- Microsoft Excel: a popular spreadsheet program used for organizing and analyzing data. We used Excel to manipulate and organize our data sets, including the GPS coordinates and the results of our area calculations.
- Python: a well-known and popular programming language. Used for data processing and triangulation calculations.
- Blender: an open-source 3D computer software tool for modeling, rigging, animation and much more. Used to create the actual 3D model of the lake.

Scan for Lake Model